

Affleck-Dine condensate, late thermalization and the gravitino problem

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In this clarifying note we discuss the late decay of an Affleck-Dine condensate by providing a no-go theorem that attributes to conserved global charges which are identified by the net particle number in fields which are included in the flat direction(s). For a rotating condensate, this implies that: (1) the net baryon/lepton number density stored in the condensate is always conserved, and (2) the total particle number density in the condensate cannot decrease. This reiterates that, irrespective of possible non-perturbative particle production due to D -terms in a multiple flat direction case, the prime decay mode of an Affleck-Dine condensate will be perturbative as originally envisaged. As a result, cosmological consequences of flat directions such as delayed thermalization as a novel solution to the gravitino overproduction problem will remain virtually intact.

I. INTRODUCTION

The scalar potential of the Minimal Supersymmetric Standard Model (MSSM) has many flat directions [1]. These directions are classified by *gauge-invariant monomials* of the theory, and most of them carry baryon and/or lepton number [2, 3]. The flat directions have many important consequences for the early universe cosmology [1]. Most notably, there are two flat directions which can potentially act as the inflaton and can be tested at the LHC [4] (see also [5])¹.

Moreover it is well known that a baryon/lepton carrying flat direction can generate the observed baryon asymmetry via Affleck-Dine mechanism [10]. During inflation a condensate is formed along the flat direction. After inflation, the condensate starts rotating once the Hubble rate drops below its mass. This results in a baryon/lepton asymmetry which will be transferred to fermions upon the decay of the condensate. The Vacuum Expectation Value (VEV) of the condensate induces large masses to the fields which are coupled to it. The decay to these fields will be only possible when the condensate VEV has been redshifted to sufficiently small values. This will result in a late perturbative decay of the flat direction condensate [10]. A late decay of an Affleck-Dine condensate has another important consequence in a supersymmetric universe, namely late thermalization of the inflaton decay products [11, 12]. The flat direction VEV breaks the Standard Model (SM) gauge symmetry, thus inducing large masses to gauge (and gaugino) fields via the Higgs mechanism. This will slow down thermalization by suppressing dominant reactions which establish kinetic and chemical equilibrium among the inflaton decay products². A delayed thermalization results in

a reheating temperature much lower than what usually thought. This naturally solves the outstanding problem of thermal gravitino overproduction in supersymmetric models [11]³.

The aim of the note is to underline the crucial importance of conserved global charges, which was first observed in seminal papers by Affleck and Dine [10], and by Dine, Randall and Thomas [2]. Charges identified by the net particle number in fields which are included in a flat direction, most notably baryon and lepton number, are preserved by the D -terms⁴. For a (maximally) rotating condensate, this implies that possible non-perturbative effects cannot change the baryon/lepton number density stored in the condensate, and will not decrease the total number density of quanta in the condensate. As we will briefly mention, under general circumstances, this also holds when F -terms are taken into account. The decay of a rotating condensate into other degrees of freedom happens through the F -term couplings. As discussed in the original work of Affleck and Dine [10], this decay occurs late and is perturbative⁵. This guarantees that cosmological consequences of an Affleck-Dine condensate, such as late thermalization, will proceed naturally.

Similar conclusion arises for multiple flat directions represented by a *gauge-invariant polynomial* (for a detailed discussion, see Ref. [18, 25]), as it is just a manifestation of the conservation of global charges carried by a rotating condensate.

discussed in [13, 14].

³ Thermalization in the presence of supersymmetric flat directions was first considered in [15] (for some of the works in the non-supersymmetric case, see [16]). The effect of Q -balls formed from fragmentation of flat direction oscillations on reheating is discussed in [17].

⁴ The A -term does not preserve these charges. However it becomes irrelevant after the very first oscillations, as it is redshifted away rapidly [2, 19].

⁵ It is known that the F -terms cannot lead to a non-perturbative decay of a rotating condensate [24].

¹ Other important applications of flat directions include curvaton mechanism [6], inhomogeneous reheating and density perturbations [7], magnetic field generation [8], and non-thermal dark matter [9].

² Finite temperature effects on MSSM flat directions have been

II. ROTATING FLAT DIRECTIONS

A. Brief introduction to flat directions

The scalar potential of the MSSM has a large number of flat directions. The D -term and F -term contributions to the potential identically vanish along these directions. The D -flat directions are categorized by gauge-invariant combinations of the MSSM (super)fields Φ_i . The D -flatness requires that ⁶

$$\sum_i \Phi_i^* T^a \Phi_i = 0 \quad (1)$$

$$\sum_i q_i |\Phi_i|^2 = 0. \quad (2)$$

T^a are generators of the $SU(3)_c$ and $SU(2)_L$ symmetries, and q_i are the charges of Φ_i under $U(1)_Y$.

A subset of D -flat directions are also F -flat, in a sense that the superpotential makes no contribution to the potential at the renormalizable level. However the flat directions are lifted by supersymmetry breaking terms, and non-renormalizable superpotential terms induced by physics beyond the standard model. Hence the potential along a flat direction, denoted by ϕ , generically follows

$$V(\phi) = m^2 |\phi|^2 + \lambda^2 \frac{|\phi|^{2(n-1)}}{M^{2(n-3)}} + \left(A\lambda \frac{\phi^n}{M^{n-3}} + \text{h.c.} \right). \quad (3)$$

Here m , $A \sim \mathcal{O}(\text{TeV})$ are the soft supersymmetry breaking mass and A -term respectively. M is a high scale where new physics appears (like M_P or M_{GUT}), $n > 4$, and $\lambda \sim \mathcal{O}(1)$ typically.

The flat direction field ϕ acquires a large VEV during inflation as a result of the accumulation of quantum fluctuations in that epoch. This leads to the formation of a condensate along the flat direction [1] ⁷. After inflation the VEV of the condensate slowly rolls down. This continues until the time when the Hubble expansion rate is $\simeq m$, see Eq. (3). The VEV of the condensate at this time is $\varphi_0 \sim (mM^{n-3})^{1/n-2} \gg m$ [2], and all three terms in Eq. (3) are comparable in size.

Of particular importance is the A -term which, by exerting a torque, results in the rotation of the condensate.

The rotation builds up already within the first Hubble time [2]. The A -term is quickly redshifted compared with the mass term as it is higher in order (i.e. is a non-renormalizable term). It becomes negligible after a few Hubble times, thus leading to a freely rotating condensate. The trajectory of the motion in the ϕ plane is an ellipse, and its eccentricity is $\simeq 1$ as the A -term is initially as large as the mass term [2]. For our purpose, it can be approximated with a circular trajectory:

$$\phi_R = \varphi \cos(mt) \quad \phi_I = \varphi \sin(mt), \quad (4)$$

where φ is redshifted $\propto a^{-1}$ due to Hubble expansion (a being the scale factor of the universe).

B. Physical degrees of freedom

Let us consider the simplest flat direction represented by the $H_u L$ gauge-invariant combination. Here H_u is the Higgs doublet which gives mass to the up-type quarks, and L is a left-handed (LH) lepton doublet ⁸. After imposing the D -flatness condition in Eqs. (1,2), one can always go to a basis where the complex scalar field (the superscripts denote the weak isospin components of the doublets and R , I denote the real and imaginary parts of a scalar field respectively)

$$\phi = \frac{(H_u^2 + L^1)}{\sqrt{2}}, \quad (5)$$

represents a flat direction. The Vacuum Expectation Value (VEV) of ϕ , denoted by φ , breaks the $SU(2)_L \times U(1)_Y$ down to $U(1)_{\text{em}}$ (in exactly the same fashion as in the electroweak symmetry breaking). The three gauge bosons of the broken subgroup then obtain masses $\sim g\varphi$ (g denotes a general gauge coupling).

After making the following definitions:

$$\begin{aligned} \chi_1 &= \frac{H_u^2 - L^1}{\sqrt{2}} \\ \chi_2 &= \frac{H_u^1 + L^{2*}}{\sqrt{2}} \\ \chi_3 &= \frac{H_u^1 - L^{2*}}{\sqrt{2}}, \end{aligned} \quad (6)$$

we find the instantaneous mass eigenstates

$$\begin{aligned} \chi'_1(t) &= \frac{\cos(mt)\chi_{1,R} + \sin(mt)\chi_{1,I}}{\sqrt{2}} \\ \chi'_2(t) &= \frac{\cos(mt)\chi_2 + \sin(mt)\chi_3}{\sqrt{2}}, \end{aligned} \quad (7)$$

⁶ Here we use the same symbol for a superfield and its scalar component.

⁷ The supergravity corrections to the flat direction potential are given by $cH_{\text{inf}}^2 |\phi|^2$ [2] (H_{inf} being the Hubble expansion rate during inflation). If $0 < c \sim \mathcal{O}(1)$, the flat direction will settle at the origin during inflation and will play no dynamical role afterwards. However, if $0 < c \ll 1$, or for $c < 0$, the flat direction acquires a large VEV during inflation and plays an important role in the post-inflationary universe. We will be mainly concerned with the latter case. Note that if there is a positive $\mathcal{O}(1)$ Hubble correction to the flat direction mass, then it would also be reflected in the inflationary potential which would spoil the success of inflation [1].

⁸ The situation will be similar for the $H_u H_d$ flat direction.

which acquire masses equal to those of the gauge bosons through the D -term part of the scalar potential (χ'_2 has both real and imaginary parts). Note that

$$\begin{aligned}\chi'_3(t) &= \frac{\cos(mt)\chi_{,I}^1 - \sin(mt)\chi_{,R}^1}{\sqrt{2}} \\ \chi'_4(t) &= \frac{\cos(mt)\chi_3 - \sin(mt)\chi_2}{\sqrt{2}},\end{aligned}\quad (8)$$

are the three Goldstone bosons (again χ'_4 has both real and imaginary parts), which are eaten up by the massive gauge fields via the Higgs mechanism. Therefore, out of the 8 real degrees of freedom in the two doublets, there are only two *physical* light fields: ϕ_R and ϕ_I , i.e. the real and imaginary parts of the flat direction field⁹.

A rotating flat direction, see Eq. (4), does not cross the origin. Hence, starting with a large VEV such that $g\varphi \gg m$, the hierarchy between the mass eigenvalues of the heavy and the light degrees of freedom is preserved at all times. However, rotation results in time variation in the mass eigenstates of the fields Eq. (7,8).

The heavy fields, despite having time-varying mass eigenstates (7), evolve adiabatically at all times since $g\varphi \gg m$, and hence will not experience any non-perturbative effects. In fact, they get decoupled and become dynamically irrelevant.

If there are light fields with a mass $< m$, time variation will become non-adiabatic and can lead to non-perturbative production of their quanta [20, 21, 22, 23]. In the case at hand the only physical light degrees of freedom, i.e. degrees of freedom on the D -flat subspace after removing the Goldstone modes (8), are the real and imaginary parts of the flat direction field (5). The mass eigenstates and eigenvalues of the flat direction are both constant in time, therefore, there will be no non-perturbative effects whatsoever (for detailed discussion, see Ref. [18]).

In general, the number of physical degrees of freedom on the D -flat subspace is given by [18]

$$N_{\text{light}} = N_{\text{total}} - (2 \times N_{\text{broken}}). \quad (9)$$

N_{total} is the total number of degrees of freedom in the gauge-invariant combination which represents the flat direction(s), and N_{broken} is the number of spontaneously broken symmetries. The factor 2 counts for the Goldstone bosons which are eaten by the gauge fields plus the superheavy degrees of freedom which are decoupled.

In the case of $H_u L$ flat direction, Eq. (9) reads: $N_{\text{light}} = (2 \times 2 \times 2) - (2 \times 3) = 2$, as explicitly shown above.

⁹ The masses induced by the flat direction VEV are supersymmetry conserving. One therefore finds the same mass spectrum in the fermionic sector as in above (for details, see [18]). Note however that scalars also acquire soft supersymmetry breaking masses $\sim \mathcal{O}(\text{TeV})$, while fermions do not. This implies that the fermionic partner of the flat direction field ϕ will remain exactly massless.

This is the typical tendency for a single flat direction in MSSM [18], i.e. when the flat direction is represented by a gauge-invariant *monomial*. Therefore particle production due to rotation can only be possible if one considers two or more flat directions, typically represented by a gauge-invariant *polynomial* [25].

Nevertheless, we would like to emphasize that time-variation in the mass eigenstates of light fields, even if it happens, has practically no bearing for the decay of rotating flat direction(s). This is the topic which we will discuss in the next section.

III. NO-GO THEOREM FOR ROTATING FLAT DIRECTIONS

Let us consider MSSM flat direction(s) represented by a gauge-invariant combination of the fields Φ_i . The D -term part of the potential, see Eqs. (1,2), is invariant under arbitrary phase transformations of Φ_i (the same is true for kinetic terms)¹⁰:

$$\Phi_i \rightarrow e^{i\alpha_i} \Phi_i. \quad (10)$$

The associated conserved charges (“.” denotes differentiation with respect to time)

$$n_i = i\dot{\Phi}_i^* \Phi_i + \text{h.c.}, \quad (11)$$

represent the *net* particle number density, i.e. the *difference* between the number density of particles and anti-particles, in Φ_i ¹¹:

$$n_i = (n_{\text{particle}} - n_{\text{anti-particle}})_i. \quad (12)$$

The *total* particle number density in Φ_i , denoted by \tilde{n}_i , is the *sum* of particle and anti-particle number densities:

$$\tilde{n}_i = (n_{\text{particle}} + n_{\text{anti-particle}})_i \geq |n_i|. \quad (13)$$

For maximally rotating flat direction(s), the fields Φ_i follow:

$$\Phi_i = \frac{\phi_i}{\sqrt{2}} \exp(i\theta_i), \quad (14)$$

where $\dot{\phi}_i = 0$, and $\ddot{\theta}_i = 0$ from the equations of motion¹². Eq. (11) then reads

$$n_i = \dot{\theta}_i \phi_i^2. \quad (15)$$

¹⁰ Some combinations of these phases are associated with the $U(1)_Y$ and $U(1)$ subgroups from diagonal generators of $SU(3)_c$ and $SU(2)_L$. Hence in a background with non-zero flat direction VEV they correspond to Goldstone bosons which, in the unitary gauge, are completely removed from the spectrum. We will deal with this carefully in the case of explicit examples presented in the next section.

¹¹ The A -term, see Eq. (3), breaks these symmetries. However it virtually decouples within a few Hubble times after the rotation starts. In particular it will be irrelevant by the time possible non-perturbative effects become important.

¹² For a freely rotating scalar field with mass m_i we have $\dot{\theta}_i = m_i$.

On the other hand, \tilde{n}_i is given by (the factor of 2 accounts for the kinetic term and the mass term of Φ_i)

$$\tilde{n}_i = 2|\dot{\theta}_i|^{-1}|\dot{\Phi}_i|^2 = |\dot{\theta}_i| \phi_i^2. \quad (16)$$

The inequality in Eq. (13) is therefore saturated, $\tilde{n}_i = |n_i|$, for maximal rotation. This is not surprising as such a condensate consists of particles or anti-particles *only*.

This leads to a following *no-go theorem*:

Consider MSSM flat direction(s) represented by gauge-invariant combinations of fields Φ_i . Possible non-perturbative particle production from time-variation in the mass eigenstates caused by the D -terms:

(1) *cannot change the net particle number density in Φ_i , denoted by n_i , and hence the total baryon/lepton number density stored in the condensate.*

(2) *cannot decrease the total particle number density in Φ_i , denoted by \tilde{n}_i , thus the total number density of quanta $\tilde{n} = \sum_i \tilde{n}_i$ in the condensate¹³. As a direct consequence of the conservation of energy density, non-perturbative effects will not increase the average energy of quanta E_{ave} .*

We note that, so far as the D -terms are concerned, the theorem also applies to the subsequent evolution of the plasma formed after the phase of particle production. This implies that possible non-perturbative effects do not lead to the decay of a rotating condensate. They merely redistribute the energy which is initially stored in the condensate among the fields on the D -flat subspace¹⁴.

Some comments are in order before closing this section. Particle production drains energy from the rotating condensate. The question is how this energy transfer affects the flat direction(s) trajectory. The D -terms, see Eqs. (1,2), and kinetic terms are invariant under interchanging the real and imaginary components of scalar fields Φ_i which are included in the flat direction(s). For a circular motion the trajectory itself is invariant under such interchanges. This implies that possible non-perturbative particle production, which is governed by D -terms and kinetic terms, will not change the shape of a circular trajectory. Therefore all that happens in the case of maximal rotation, see Eq. (4), is a decrease in the radius of the circle φ .

In reality, the condensate will not undergo maximal rotation and trajectory of its motion will be an ellipse:

$$\phi_R = \varphi \cos(mt) \quad \phi_I = \alpha \varphi \sin(mt), \quad (17)$$

rather than a circle (4), where $\alpha < 1$ is a positive number. However, for a generic initial condition, the rotation will be near maximal $\alpha \sim 0.3$ [2]. The difference between the number density of particles and antiparticles in the condensate is given by

$$n = 2\alpha m \varphi^2, \quad (18)$$

while the total number density of quanta in the condensate follows

$$\tilde{n} = (1 + \alpha^2)m\varphi^2. \quad (19)$$

Note that for $\alpha = 1$ the trajectory is a circle and $n = \tilde{n}$ as we discussed above. However, for $\alpha \sim 1$ we have $n \sim \tilde{n}$ (for $\alpha < 1$ we always have $\tilde{n} > n$). The condensate in this case consists mainly of particles (or anti-particles), but it *also* contains a small mixture of anti-particles (or particles). Therefore, in agreement with the conservation of net particle number density, \tilde{n} can in principle decrease by a factor of $r = (1 + \alpha^2)/2\alpha$, such that the small mixture of (anti-)particles will vanish and consequently $\tilde{n} = n$. For $\alpha \sim 0.3$, we have $r \sim 2$. The possible decrease in \tilde{n} will therefore be of $\mathcal{O}(1)$. As we will discuss later on, the situation is similar to that for a maximal rotation with regard to the final decay of the flat direction(s) energy density and late thermalization of the universe.

Finally, we note that the F -terms, due to quark and lepton mixing, preserve $\sum_i n_i$ (i.e. the total baryon/lepton number density) instead of each n_i . However, $|\sum_i n_i| \sim \sum_i |n_i|$, except in some cases where $\sum_i n_i$ is much smaller than the individual n_i . This requires special initial conditions for which the baryon/lepton number density stored in individual fields are large, but come with opposite signs in such a way that they conspire to make the total baryon/lepton number density which is stored in the condensate much smaller¹⁵. Hence, under general circumstances, the no-go theorem holds when all interactions in the MSSM Lagrangian are taken into account.

IV. SOME EXAMPLES

To elucidate the no-go theorem, we consider three representative examples of MSSM flat directions. Namely, single flat directions consisting of two and three fields, and multiple flat directions.

¹³ Here we mean the comoving quantities as the Hubble expansion inevitably redshifts any physical number density.

¹⁴ As we will explain later, this is in sharp contrast to non-perturbative particle production from an oscillating condensate, also called preheating, studied in the context of inflaton decay [26].

¹⁵ The only case where this can happen naturally is for the $H_u H_d$ flat direction which carries $B = L = 0$. However, this is a single flat direction for which there is no time variation in the light physical degrees of freedom [18]. Therefore there can be no non-perturbative particle production in this case at the first place.

A. $H_u L$ direction

The D -terms associated with $SU(2)_L$ and $U(1)_Y$, see Eqs. (1,2), are invariant under two $U(1)$ symmetries:

$$H_u \rightarrow e^{i\alpha_1} H_u, \quad L \rightarrow e^{i\alpha_2} L, \quad (20)$$

and the corresponding charges

$$n_1 = i\dot{H}_u^* H_u + \text{h.c.}, \quad n_2 = i\dot{L}^* L + \text{h.c.}, \quad (21)$$

are conserved.

In a background of rotating flat direction, transformations generated by non-diagonal generators of $SU(2)_L$ can be used to situate the VEVs along H_u^2 and L^1 (superscripts denote the weak isospin components), which we denote by ϕ_1 and ϕ_2 respectively:

$$\phi_1 = \frac{\varphi}{2} \exp(i\theta_1), \quad \phi_2 = \frac{\varphi}{2} \exp(i\theta_2). \quad (22)$$

The phase difference $\theta_2 - \theta_1$ is a Goldstone boson which can be removed through a $U(1)_Y$ transformation¹⁶ (for identification of Goldstone bosons, see Section IX). This, as shown before, leaves us with only two light degrees of freedom

$$\phi_1 = \frac{\varphi}{2} \exp(i\theta), \quad \phi_2 = \frac{\varphi}{2} \exp(i\theta). \quad (23)$$

Eq. (21) then results in

$$n_1 = n_2 = \dot{\theta} \varphi^2. \quad (24)$$

For a rotating flat direction $\dot{\varphi} = 0$. Then from the equations of motion we find $\dot{\theta}^2 = (m_H^2 + m_L^2)/2$, where m_H and m_L are the masses of H and L respectively. Note that n_2 is the lepton number density stored in the condensate.

The total particle number density in H_u and L (denoted by \tilde{n}_1 and \tilde{n}_2 respectively) follow from Eq. (16):

$$\begin{aligned} \tilde{n}_1 &= |\dot{\theta}| \varphi^2 = |n_1|, \\ \tilde{n}_2 &= |\dot{\theta}| \varphi^2 = |n_2|. \end{aligned} \quad (25)$$

B. udd and LLe directions

The situation for udd and LLe flat directions is quite similar. We therefore concentrate on the udd case. The D -terms associated with $SU(3)_c$ and $U(1)_Y$, see Eqs. (1,2), are invariant under three $U(1)$ symmetries (subscripts are the family indices):

$$u_i \rightarrow e^{i\alpha_{i1}} u_i, \quad d_j \rightarrow e^{i\alpha_{j2}} d_j, \quad d_k \rightarrow e^{i\alpha_{k3}} d_k, \quad (26)$$

and the corresponding charges

$$n_1 = i\dot{u}_i^* u_i + \text{h.c.}, \quad n_2 = i\dot{d}_j^* d_j + \text{h.c.}, \quad n_3 = i\dot{d}_k^* d_k + \text{h.c.}, \quad (27)$$

are conserved.

In a rotating flat direction background, transformations generated by non-diagonal generators of $SU(3)_c$ can be used to situate the VEVs along u_i^1 , d_j^2 , d_k^3 (which we denote by ϕ_1 , ϕ_2 , ϕ_3 respectively), where superscripts denote the color indices:

$$\begin{aligned} \phi_1 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta_1), \\ \phi_2 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta_2), \\ \phi_3 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta_3). \end{aligned} \quad (28)$$

The phase differences $(2\theta_1 - \theta_2 - \theta_3)$ and $\theta_1 - \theta_2$ are Goldstone modes which can be removed through transformations generated by diagonal generators of $SU(3)_c$ ¹⁷ (for identification of Goldstone bosons, see Section XI). After the removal of Goldstone bosons, only two light degrees of freedom remain¹⁸

$$\begin{aligned} \phi_1 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta), \\ \phi_2 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta), \\ \phi_3 &= \frac{\varphi}{\sqrt{6}} \exp(i\theta). \end{aligned} \quad (29)$$

For a rotating flat direction we have $\dot{\varphi} = 0$. Then from the equations of motion we find $\dot{\theta}^2 = (m_{u_i}^2 + m_{d_j}^2 + m_{d_k}^2)/3$. Eq. (27) results in

$$n_1 = n_2 = n_3 = \dot{\theta} \varphi^2. \quad (30)$$

Note that $n = n_1 + n_2 + n_3$ is three times the baryon number density stored in the rotating condensate (u and d have baryon number $-1/3$).

The total particle number density in u_i , d_j , d_k (denoted by \tilde{n}_1 , \tilde{n}_2 , \tilde{n}_3 respectively) follow from Eq. (16):

$$\begin{aligned} \tilde{n}_1 &= |\dot{\theta}| \varphi^2 = |n_1|, \\ \tilde{n}_2 &= |\dot{\theta}| \varphi^2 = |n_2|, \\ \tilde{n}_3 &= |\dot{\theta}| \varphi^2 = |n_3|. \end{aligned} \quad (31)$$

¹⁷ The action of $U(1)_Y$ is the same as that of the $(-1, -1, +2)$ diagonal generator of $SU(3)_c$.

¹⁸ This is different from the toy example presented in Ref. [22], which considers a flat direction consisting of three fields charged under a single $U(1)$ gauge symmetry. In the case of MSSM, there are enough symmetries to rotate away all phase differences among the fields, and hence only the overall phase remains as a physical degree of freedom.

¹⁶ Or, equivalently, the diagonal generator of $SU(2)_L$.

C. $\sum_i H_u L_i$ multiple flat directions

Now we consider multiple flat directions represented by the $\sum_{i=1}^3 H_u L_i$ polynomial where all three L_i doublets have a non-zero VEV. This case was first considered in Ref. [8]. The D -terms associated with $SU(2)_L$ and $U(1)_Y$, see Eqs. (1,2), are invariant under four $U(1)$ symmetries:

$$\begin{aligned} L_1 &\rightarrow e^{i\alpha_1} L_1, \quad L_2 \rightarrow e^{i\alpha_2} L_2, \quad L_3 \rightarrow e^{i\alpha_3} L_3 \\ H_u &\rightarrow e^{i\alpha_4} H_u, \end{aligned} \quad (32)$$

and the corresponding charges

$$\begin{aligned} n_1 &= i\dot{L}_1^* L_1 + \text{h.c.} \quad n_2 = i\dot{L}_2^* L_2 + \text{h.c.} \quad n_3 = i\dot{L}_3^* L_3 + \text{h.c.}, \\ n_4 &= i\dot{H}_u^* H_u + \text{h.c.}, \end{aligned} \quad (33)$$

are conserved.

Similar to the case of the $H_u L$ single flat direction, the VEVs can be situated along the $L_1^1, L_2^1, L_3^1, H_u^2$ components which we denote by $\phi_1, \phi_2, \phi_3, \phi_4$ respectively¹⁹:

$$\begin{aligned} \phi_1 &= \frac{\varphi_1}{2} \exp(i\theta_1), \\ \phi_2 &= \frac{\varphi_2}{2} \exp(i\theta_2), \\ \phi_3 &= \frac{\varphi_3}{2} \exp(i\theta_3), \\ \phi_4 &= \frac{\varphi}{2} \exp(i\theta_4), \end{aligned} \quad (34)$$

where $\varphi^2 = \varphi_1^2 + \varphi_2^2 + \varphi_3^2$ is imposed by the D -flatness condition, see Eqs. (1,2). The phase $(\varphi\theta - \varphi_1\theta_1 - \varphi_2\theta_2 - \varphi_3\theta_3)$ is a Goldstone mode which can be removed by a $U(1)_Y$ transformation²⁰ (for identification of Goldstone bosons, see Section X). After its removal we can recast Eq. (34) in the following form²¹:

$$\begin{aligned} \phi_1 &= \frac{\varphi_1}{2} \exp(i\theta_1), \\ \phi_2 &= \frac{\varphi_2}{2} \exp(i\theta_2), \\ \phi_3 &= \frac{\varphi_3}{2} \exp(i\theta_3), \\ \phi_4 &= \frac{\varphi}{2} \exp\left[i\left(\frac{\varphi_1\theta_1 + \varphi_2\theta_2 + \varphi_3\theta_3}{\varphi}\right)\right]. \end{aligned} \quad (35)$$

Eq. (33) now results in

$$\begin{aligned} n_1 &= \dot{\theta}_1 \varphi_1^2, \\ n_2 &= \dot{\theta}_2 \varphi_2^2, \\ n_3 &= \dot{\theta}_3 \varphi_3^2, \\ n_4 &= (\varphi_1\dot{\theta}_1 + \varphi_2\dot{\theta}_2 + \varphi_3\dot{\theta}_3) \varphi. \end{aligned} \quad (36)$$

For maximal rotation $\varphi_1, \varphi_2, \varphi_3$ are constant, and $\ddot{\theta}_1 = \ddot{\theta}_2 = \ddot{\theta}_3 = 0$ from the equations of motion²².

Note that $n = n_1 + n_2 + n_3$ is the lepton number stored in the condensate. The total particle number density in L_1, L_2, L_3, H_u (denoted by $\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4$ respectively) follow from Eq. (16):

$$\begin{aligned} \tilde{n}_1 &= |\dot{\theta}_1| \varphi_1^2 = |n_1|, \\ \tilde{n}_2 &= |\dot{\theta}_2| \varphi_2^2 = |n_2|, \\ \tilde{n}_3 &= |\dot{\theta}_3| \varphi_3^2 = |n_3|, \\ \tilde{n}_4 &= |\varphi_1\dot{\theta}_1 + \varphi_2\dot{\theta}_2 + \varphi_3\dot{\theta}_3| \varphi = |n_4|. \end{aligned} \quad (37)$$

V. DIFFERENCES BETWEEN ROTATING AND OSCILLATING CONDENSATES

Let us consider an oscillating condensate for which the trajectory of motion is a line instead of a circle:

$$\phi_R = \varphi \cos(mt), \quad \phi_I = 0, \quad (38)$$

where $g\varphi \gg m$ (g is a typical gauge coupling). In this case the mass eigenstate of the χ fields which are coupled to ϕ through the D -terms, see Eq. (6), are constant in time but the mass eigenvalues oscillate. The time variation becomes non-adiabatic as $\phi \approx 0$. As a result, χ quanta are created within short intervals each time that ϕ crosses the origin [26]. This leads to an explosive stage of particle production, also called preheating, which eventually results in a *plasma* of χ and ϕ quanta with typical energy

$$E_{\text{ave}} \sim (g\varphi m)^{1/2} \gg m. \quad (39)$$

This implies an increase in the average energy of quanta, and hence a decrease in the number density of quanta, as compared to the original condensate. If the universe were to fully thermalize after preheating, we would have $E_{\text{ave}} \sim T \sim (m\varphi)^{1/2}$. Preheating is therefore a step toward full thermal equilibrium as it partially increases E_{ave} toward its equilibrium value²³.

This is in sharp contrast to the situation for a rotating condensate. There, as we argued, possible particle production cannot decrease the number density of quanta. The marked difference between the two cases can be understood from the trajectory of motion (i.e. circular for rotation versus linear for oscillation). An oscillating condensate ϕ can be written as

$$\phi = \frac{\varphi}{2} \exp(i\theta) + \frac{\varphi}{2} \exp(-i\theta), \quad (40)$$

¹⁹ The situation is actually more subtle than that for the $H_u L$ single direction, see Section IX.

²⁰ Or, equivalently, a transformation generated by the diagonal generator of $SU(2)_L$.

²¹ Note that if any two of $\varphi_1, \varphi_2, \varphi_3$ are zero, the situation will be reduced to that for the $H_u L$ single flat direction in Eq. (23).

²² The situation is more complicated than the previous examples due to having more than one physical phase. In particular, ϕ_i can rotate with different velocities, where the velocity of rotation is given by $\dot{\theta}_i$.

²³ In reality, it takes a much longer time to establish full thermal equilibrium after preheating [27].

and the conserved charge associated with the global $U(1)$ (corresponding to phase θ) is given by

$$n = i\dot{\phi}^*\phi + \text{h.c.} = 0. \quad (41)$$

This is not surprising since an oscillation is the superposition of two rotations in opposite directions, which carry exactly the same number of particles and anti-particles respectively. Therefore the net particle number density stored in an oscillating condensate is zero.

Now consider non-perturbative particle production from an oscillating condensate. One can think of this process as a series of annihilations among N particles and N anti-particles in the condensate, $N > 1$, into an energetic particle-anti-particle pair. This is totally compatible with conservation of charge, see Eq. (41); $n = 0$ after preheating as well as in the condensate.

On the other hand, a (maximally) rotating condensate consists of particles or anti-particles *only*, see Eqs. (25, 31, 37). Conservation of the net particle number density then implies that $N \rightarrow 2$ annihilations ($N > 2$) are forbidden: annihilation of particle (or anti-particle) quanta cannot happen without violating the net particle number density. Therefore the total number density of quanta will not decrease, and the average energy will not increase²⁴.

VI. DECAY OF A ROTATING CONDENSATE

As we have discussed, any possible non-perturbative particle production will result in a plasma which is at least as dense as the initial condensate. All that can happen is a redistribution of the energy density in the condensate among the fields on the D -flat subspace. These fields have masses comparable to the flat direction mass m , as they all arise from supersymmetry breaking. Then, since the average energy is $E_{\text{ave}} \leq m$, the resulting plasma essentially consists of non-relativistic quanta. Its energy density $\rho = \tilde{n}E_{\text{ave}}$ is therefore redshifted $\propto a^{-3}$ (a is the scale factor of the universe).

The question is when this plasma will decay to *other MSSM fields*, in particular fermions, and thermalize. The plasma induces a large mass m_{eff} to the scalars which are not on the D -flat subspace and their fermionic partners through the F -terms. In the Hartree approximation the effective mass is given by (for example, see [28])

$$m_{\text{eff}}^2 \sim h^2 \frac{\tilde{n}}{E_{\text{ave}}}, \quad (42)$$

²⁴ Note that an increase in the total particle number density, through creation of an equal number of particles and anti-particles will be in agreement with the conservation of the net particle number density. In this case the resulting plasma will be even denser than the condensate.

where h denotes a Yukawa coupling²⁵. The one-particle decay is kinematically forbidden as long as $m_{\text{eff}} \geq m$. Note that higher order processes such as $N \rightarrow 2$ annihilations ($N > 2$) cannot happen due to conservation of global charges (i.e. baryon and lepton number) in the plasma. Since \tilde{n} and E_{ave} are respectively \geq and \leq than their corresponding values in the initial condensate, thus m_{eff} will always be larger than the induced mass by the condensate VEV, which is given by $h\varphi$.

Further note that m_{eff} is redshifted $\propto a^{-3/2}$, where $a \propto H^{-2/3}$ ($H^{-1/2}$) in a matter (radiation) dominated epoch. The decay of energy density ρ (initially stored in the condensate) happens *only* when m_{eff} has been redshifted below m , at which time the Hubble expansion rate is given by (φ_0 is the initial VEV of the condensate) [11, 12]

$$\begin{aligned} H_{\text{dec}} &\sim m \left(\frac{m}{h\varphi_0} \right) && \text{(matter domination),} \\ H_{\text{dec}} &\sim m \left(\frac{m}{h\varphi_0} \right)^{4/3} && \text{(radiation domination).} \end{aligned} \quad (43)$$

Hence for a large φ_0 the decay time scale is sufficiently large compared to the time scale for possible non-perturbative particle production, which is $\sim m^{-1}$. This decay happens perturbatively as discussed in [2, 10].

As we discussed earlier, in reality the condensate has an elliptic trajectory whose eccentricity is ~ 0.3 . This, see Eqs. (18,19), implies that \tilde{n} can at most decrease (and hence E_{ave} increase) by a factor of 2 compared with their corresponding values in the initial condensate. Therefore m_{eff} , see Eq. (42), may be smaller by a factor of 2 in the case of a realistic elliptic trajectory. According to Eq. (43), this will result in an H_{dec} which is larger by a similar factor, thus a slightly earlier perturbative decay. Nevertheless, for $\varphi_0 \gg m$, the final decay happens much later than the initial phase of non-perturbative particle production.

This reiterates the main point of this note: a phase of non-perturbative particle production due to rotation, although possible, cannot lead to the decay of flat direction energy density. It will merely result in a redistribution of energy density on the D -flat subspace. The final decay (to other fields) will happen late, and will be perturbative, as originally envisaged [2, 10].

VII. COSMOLOGICAL CONSEQUENCES

For practical purposes, the resulting plasma will behave the same as the initial condensate. In this section we discuss some of the important cosmological consequences for an Affleck-Dine condensate which gives rise to delayed thermalization and a solution to the gravitino problem.

²⁵ In the case of thermal equilibrium (and zero chemical potential) we have $\tilde{n} \sim T^3$ and $E_{\text{ave}} \sim T$, where T is the temperature, yielding the familiar result $m_{\text{eff}}^2 \sim h^2 T^2$.

A. Delayed thermalization

The condensate VEV, denoted by φ , spontaneously breaks the SM gauge symmetry and induces a large mass for the gauge/gaugino fields $m_{\text{eff}} \sim g\varphi$ via the Higgs mechanism. Such a large mass suppresses gauge interactions which play the main role in establishing thermal equilibrium among inflaton decay products [11]. For a rotating condensate φ changes only due to the Hubble redshift. The gauge interactions will therefore remain ineffective for a long time until φ has been redshifted to a sufficiently small value. It is only at this time that full thermal equilibrium can be established [11, 12].

Now consider the plasma consisting of quanta of the fields Φ_i on the D -flat subspace. The SM gauge symmetry is broken in the presence of this plasma as well. This results in an induced mass m_{eff} for the gauge fields (and gauginos) which, as mentioned earlier, is given by

$$m_{\text{eff}}^2 \sim g^2 \frac{\tilde{n}}{E_{\text{ave}}}. \quad (44)$$

Since n and E_{ave} are respectively \geq and \leq than the corresponding values in the initial condensate, it turns out that $m_{\text{eff}} \geq g\varphi$. This implies that the gauge interactions will be (at least) as suppressed as that in the presence of a condensate. Hence, considering that the plasma decays like the initial condensate, thermalization will also be delayed similarly, for details see [11, 12].

Again we note that for a realistic elliptic trajectory m_{eff} may be smaller by a factor of 2. However, for $\varphi_0 \gg m$, universe thermalization will still be considerably delayed relative to an initial phase of non-perturbative particle production from the rotating flat direction(s).

B. Thermal generation of gravitinos

Late thermalization has an important consequence for thermal production of gravitinos [29]²⁶.

First, delayed thermalization leads to a considerably low reheat temperature given by the expression [11]

$$T_R \sim (\Gamma_{\text{thr}} M_P)^{1/2}, \quad (45)$$

instead of the usual expression $T_R \sim (\Gamma_d M_P)^{1/2}$. Here Γ_{thr} is the rate for thermalization of the inflaton decay products and Γ_d is the inflaton decay rate. Suppression of the interactions that lead to establishment of thermal equilibrium, due to the VEV of flat direction(s), implies that $\Gamma_{\text{thr}} \ll \Gamma_d$ [11], and hence a much lower T_R than usually found.

In particular, one can naturally obtain $T_R \ll 10^9$ GeV which, in the case of a weak scale supersymmetry, is required in order not to distort predictions of the Big Bang Nucleosynthesis by the decay of gravitinos [29].

Moreover, before thermalization of the inflaton decay products, scattering processes which lead to gravitino production make a negligible contribution (for details, see [11]). These two effects address the long standing gravitino problem in a natural way within supersymmetry without invoking any ad-hoc mechanism.

VIII. CONCLUSION

The important message of this note is that possible non-perturbative effects stemmed from the D -terms have no bearing for the decay of energy density in rotating flat direction(s). This is due to the conservation of global charges associated with the net particle number density in fields which are included in the flat direction(s), most notably the baryon/lepton number density [2, 10]. For a rotating condensate, this ensures that the total number density of quanta will not decrease and, consequently, the average energy of quanta will not increase.

Thus, in sharp contrast to an oscillating condensate (as in the case of inflaton decay via preheating), all that can happen is a mere redistribution of the condensate energy among the fields on the D -flat subspace. The actual decay into other fields happens perturbatively as originally envisaged by Affleck and Dine [10]. This ensures the success of cosmological consequences such as delayed thermalization as a novel solution to the gravitino problem [11, 12].

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X. IDENTIFICATION OF GOLDSTONE BOSONS

Here we quickly comment on identification of the Goldstone bosons and their removal from the spectrum. For simplicity, we consider the case with a single $U(1)$ gauge symmetry, but generalization to non-abelian symmetries is straightforward.

Consider n scalar fields ϕ_i with respective charges q_i ($1 \leq i \leq n$) under the $U(1)$ symmetry. Covariant deriva-

²⁶ Non-thermal gravitino production at early stages of inflaton oscillations [30] is not a major issue as discussed in [31].

tives of the scalar fields are

$$\sum_{i=1}^n (\partial^\mu + iA^\mu) \phi_i^* (\partial_\mu - iA_\mu) \phi_i. \quad (46)$$

The scalar fields can be written in terms of radial and angular components (denoted by φ and θ respectively):

$$\phi_i = \frac{\varphi_i}{\sqrt{2}} \exp(i\theta_i). \quad (47)$$

Expanding the fields around a background where φ_i is constant, as happens for rotating flat direction(s), we then have

$$\partial_\mu \phi_i = i(\partial_\mu \theta_i) \frac{\varphi_i}{\sqrt{2}} \exp(i\theta_i). \quad (48)$$

It can be seen that the combination $\sum_{i=1}^n (\varphi_i q_i \theta_i)$ can be eliminated from Eq. (46) by performing the following gauge transformation:

$$\begin{aligned} \theta_i &\rightarrow \theta_i + q_i \theta_i, \\ A_\mu &\rightarrow A_\mu - \sum_{i=1}^n q_i (\partial_\mu \theta_i). \end{aligned} \quad (49)$$

Therefore it is not a true physical degree of freedom. This particular combination is nothing but the Goldstone boson from spontaneous breaking of $U(1)$ symmetry by non-zero values of φ_i .

XI. $\sum_i H_u L_i$ MULTIPLE FLAT DIRECTIONS

Consider a general VEV configuration of H_u and L_i ($1 \leq i \leq 3$) that satisfies the D -flatness condition in

Eqs. (1,2). We can always use non-diagonal generators of $SU(2)_L$ to rotate H_u to a basis where $\langle H_u^1 \rangle = 0$ (superscripts denote the weak isospin component).

In the case of $H_u L$ single flat direction (where two of the L_i have zero VEV), D -flatness under the non-diagonal generators directly implies $\langle L^2 \rangle = 0$ in this basis. However, for multiple flat directions, it is not so obvious that $\langle L_1^2 \rangle = \langle L_2^2 \rangle = \langle L_3^2 \rangle = 0$ in the basis where $\langle H_u^1 \rangle = 0$.

Hence let us consider a general configuration where both isospin components of L_i have a non-zero VEV. The vanishing of the D -term associated with the diagonal generator of $SU(2)_L$ then implies

$$-|\langle H_u^2 \rangle|^2 + \sum_{i=1}^3 |\langle L_i^1 \rangle|^2 - \sum_{i=1}^3 |\langle L_i^2 \rangle|^2 = 0, \quad (50)$$

while vanishing of the D -term associated with $U(1)_Y$ requires that (note that H_u and L_i have opposite hypercharge quantum numbers)

$$|\langle H_u^2 \rangle|^2 - \sum_{i=1}^3 |\langle L_i^1 \rangle|^2 - \sum_{i=1}^3 |\langle L_i^2 \rangle|^2 = 0. \quad (51)$$

It is readily seen that the third term on the LH side of Eqs. (50,51) must vanish, thus $\langle L_1^2 \rangle = \langle L_2^2 \rangle = \langle L_3^2 \rangle = 0$. Also:

$$|\langle H_u^2 \rangle|^2 = \sum_{i=1}^3 |\langle L_i^1 \rangle|^2. \quad (52)$$

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